# Sample-Based Methods for Continuous Action Markov Decision Processes

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## From Learning to Planning

#### Bellman Equation

$$V(s) = \max_{a} (R(s, a) + \gamma \sum_{s'} T(s, a, s') V(s'))$$

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Continuous State Space

Standard machine learning approaches to function approximation have proven successful!

## From Learning to Planning

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Continuous Action Space

Very little work addressing how to evaluate the maximum

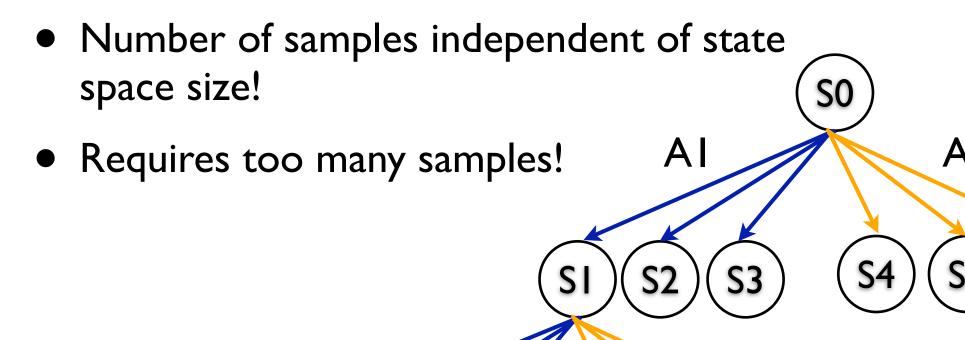
Continuous State Space

Standard machine learning approaches to function approximation have proven successful!

## Sparse Sampling

[Kearns, et al 1999]

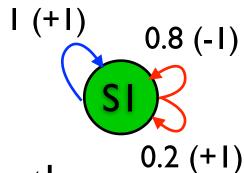
 An epsilon-optimal planning algorithm for discounted MDPs.



## Can we use ideas from the exploration/exploitation problem to better direct our search?

## **UCB**

[Auer, et al 2002]



- An algorithm for efficient learning in the bandit domain
- Fixed number of discrete actions with bounded support
- Choose an arm greedily according to the following rule:

$$\widehat{\mu}_i + \sqrt{\frac{2 \ln n}{n_i}}$$

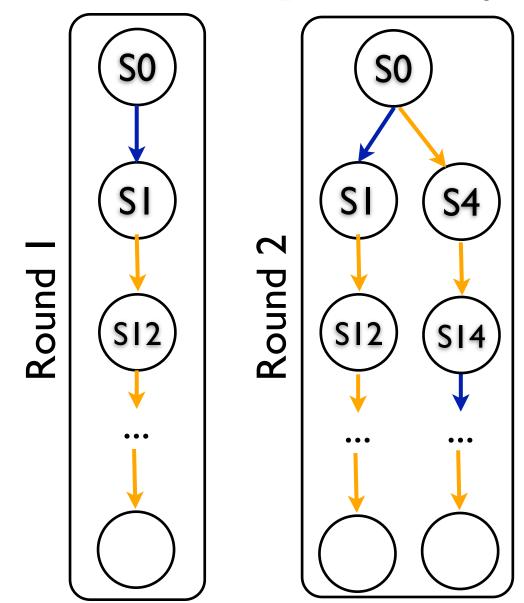
## **UCT**

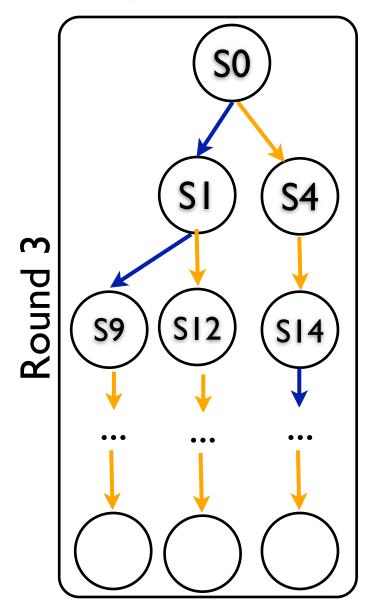
[Kocsis, Szepesvári 2006]

- Upper Confidence applied to Trees
- Takes the UCB algorithm and extends it to the full MDP domain
- Build a tree similar to SS, but instead of doing a breadth first search perform a depth first search directed by a UCB algorithm at each node

## UCT, cont...

[Kocsis, Szepesvári 2006]





## HOO

#### [Bubeck, et al 2008]

- UCT is still restricted to discrete states and actions
- HOO (hierarchical optimistic optimization) provides similar guarantees to UCB in "wellbehaved" continuous bandit problems
- The idea is simple, divide the action space up (similar to a KD-tree), keep track of returns in these volumes, provide exploration bonuses for both number of samples and size of each subdivision

## HOO, cont...

[Bubeck, et al 2008]

Choose an arm greedily with respect to the following:

$$\widehat{\mu}_i + \sqrt{\frac{2\ln n}{n_i}} + v_1 \rho^h$$

- Very similar to UCB except the spatial term at the end
- The intuition is that arms with large volumes and few samples are unknown, but small volumes and lots of samples are well known

## HOO, cont...

[Bubeck, et al 2008]

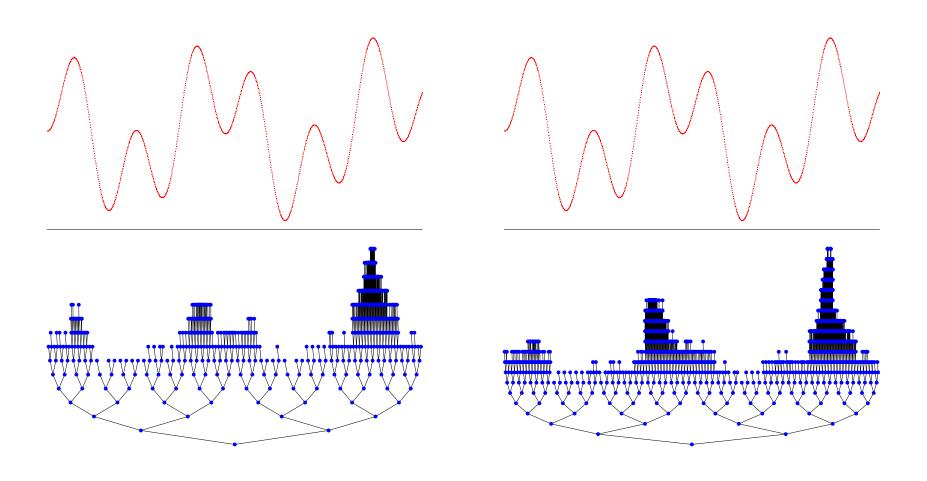
Choose an arm greedily with respect to the following:

$$\widehat{\mu}_i + \sqrt{\frac{2 \ln n}{n_i}} + \frac{\nu_1 \rho^h}{\text{diam(i)}}$$

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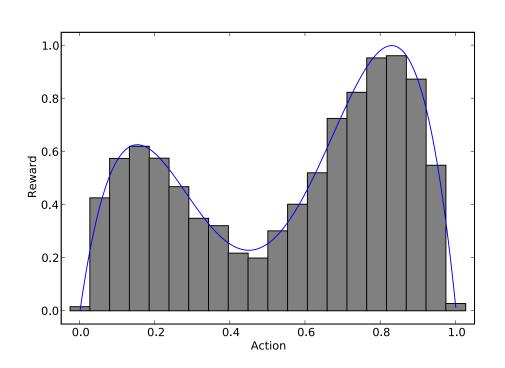
## HOO, cont...

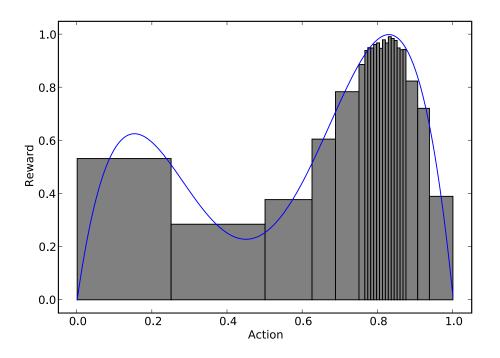
[Bubeck, et al 2008]



Thanks to Remi Munos

## UCB vs HOO

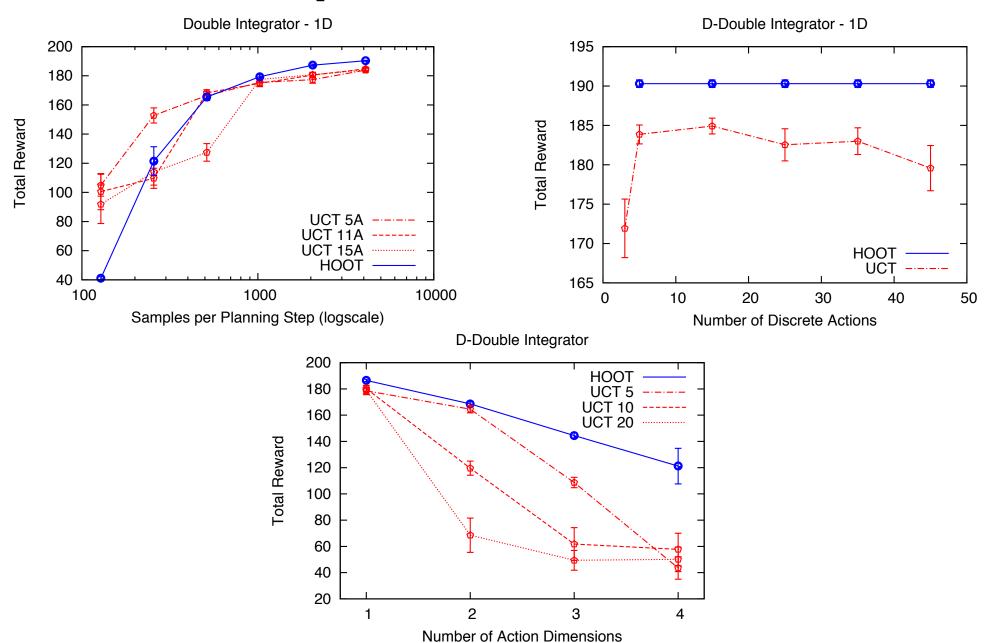




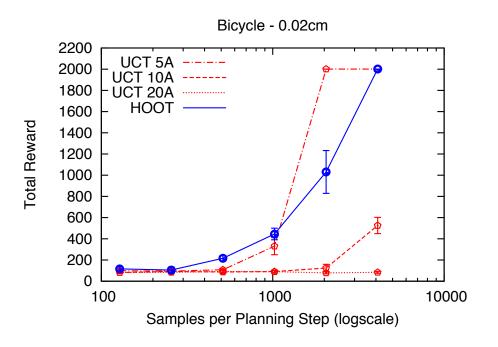
### HOOT

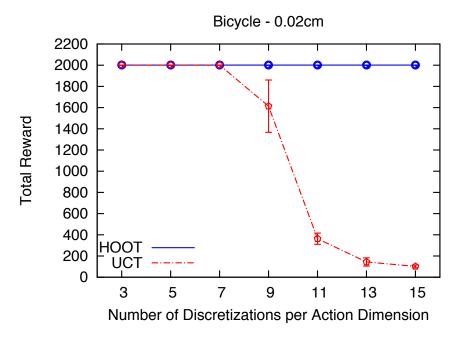
- Our idea is to replace UCB in UCT with HOO, so that we can work directly in the continuous action space
- This leads to our algorithm HOO applied to Trees (HOOT)
- The algorithm is exactly the same as UCT, but instead of using UCB at each internal node, we maintain a HOO tree

## **Empirical Results**



## Empirical Results





## Future Work

- Using HOO to optimize the n-step sequence of actions as an n-dimensional space
- Extend to continuous state spaces by a weighted interpolation between representative HOO trees

## Summary

- Choosing action discretizations is non-trival!
- If you have a distance metric and your value function is locally smooth, use HOOT not vanilla UCT!

## Thanks!